

Name: _____

Date: _____

A2CC H: Rational Exponents

If $x \geq 0$ and n is a positive integer then:

$$(1) \quad x^{\frac{1}{n}} = \sqrt[n]{x}$$

$$(2) \quad x^{\frac{m}{n}} = (\sqrt[n]{x})^m = \sqrt[n]{x^m}$$

$$(3) \quad x^{-\frac{m}{n}} = \frac{1}{(\sqrt[n]{x})^m} = \frac{1}{\sqrt[n]{x^m}}, \text{ provided that also } x \neq 0$$

Examples:

$$1. \quad 256^{\frac{1}{2}}$$

$$2. \quad 8^{\frac{1}{3}}$$

$$3. \quad \left(\frac{27}{64}\right)^{\frac{1}{3}}$$

$$4. \quad 8^{\frac{2}{3}}$$

$$5. \quad 32^{-\frac{2}{5}}$$

$$6. \quad 6^{\frac{3}{2}}$$

$$7. \quad \frac{12^0}{9^{-\frac{3}{2}}}$$

$$8 \quad \text{a. } (-27)^{\frac{1}{3}}$$

$$\text{b. } -27^{\frac{1}{3}}$$

$$9. \quad \left(\frac{-8a^3}{b^{-6}}\right)^{\frac{2}{3}}$$

Rewrite each of the following using fractional exponents.

$$10. \quad \text{a. } 5\sqrt{x}$$

$$\text{b. } \sqrt[3]{5x^2}$$

$$\text{c. } \sqrt[3]{(5x)^2}$$

$$\text{d. } \sqrt[5]{x^4y^3}$$

11. $(2 \times 8)^{\frac{1}{4}}$ 12. $5(81)^{\frac{1}{4}}$ 13. $-4(1,000)^{\frac{1}{3}}$ 14. $49^{\frac{3}{2}}$
 15. $8^{\frac{5}{3}}$ 16. $27^{\frac{4}{3}}$ 17. $10,000^{\frac{3}{4}}$ 18. $32^{\frac{4}{5}}$
 19. $9^{-\frac{1}{2}}$ 20. $8^{-\frac{1}{3}}$ 21. $100^{-\frac{3}{2}}$ 22. $125^{-\frac{2}{3}}$
 23. $3^{\frac{1}{2}} \times 3^{\frac{3}{2}}$ 24. $5^{\frac{1}{3}} \times 5^{\frac{2}{3}}$ 25. $7^{\frac{3}{4}} \times 7^{\frac{5}{4}}$ 26. $4 \times 4^{\frac{1}{2}}$
 27. $32 \times 32^{\frac{1}{2}}$ 28. $2^{\frac{1}{4}} \times 8^{\frac{1}{4}}$ 29. $12^{\frac{5}{3}} \div 12^{\frac{2}{3}}$ 30. $3^{\frac{7}{3}} \div 3^{\frac{1}{3}}$
 31. $4^{\frac{2}{3}} \div 4^{\frac{1}{6}}$ 32. $125^{\frac{2}{3}} \div 125^{\frac{1}{3}}$ 33. $4^0 + 4^{-\frac{1}{2}}$ 34. $9^{-2} + 9^{\frac{1}{2}}$
 35. $2[(3)^{-2} + (4)^{-2}]^{-\frac{1}{2}}$ 36. $(2.3 \times 10^{-1})(5.2 \times 10^{-3})$ 37. $\frac{(2(3)^2 + \frac{1}{3^{-2}})^{\frac{3}{2}}}{6(2 + \frac{1}{4})^{-\frac{1}{2}}}$

In 38–57, write each radical expression as a power with positive exponents and express the answer in simplest form. The variables are positive numbers.

38. $\sqrt{7}$ 39. $\sqrt{6}$ 40. $\sqrt[3]{12}$ 41. $\sqrt[3]{15}$
 42. $\sqrt[4]{3}$ 43. $\sqrt[5]{2^3}$ 44. $(\sqrt[5]{9})^4$ 45. $\frac{1}{(\sqrt{5})^3}$
 46. $\sqrt{25a}$ 47. $\sqrt{49x^2}$ 48. $\sqrt{64a^3b^6}$ 49. $\frac{1}{2}\sqrt{18a^6b^2}$
 50. $\sqrt{9a^{-2}b^6}$ 51. $\sqrt{\frac{3a}{4b}}$ 52. $\sqrt[3]{27a^3}$ 53. $\sqrt[4]{64x^5}$
 54. $\frac{1}{\sqrt[5]{xyz^5}}$ 55. $\sqrt{\frac{9a^{-2}}{4b^4}}$ 56. $\sqrt[10]{\frac{10^{15}x^{20}}{y^5}}$ 57. $\sqrt[8]{\sqrt[4]{a} \cdot \sqrt[4]{b^7}}$

In 58–73, write each power as a radical expression in simplest form. The variables are positive numbers.

58. $3^{\frac{1}{2}}$ 59. $5^{\frac{1}{3}}$ 60. $6^{\frac{1}{3}}$ 61. $9^{\frac{1}{3}}$
 62. $5^{\frac{3}{2}}$ 63. $12^{\frac{5}{3}}$ 64. $6^{\frac{5}{2}}$ 65. $\frac{1}{5^{\frac{1}{2}}}$
 66. $(x^{13})^{\frac{1}{4}}$ 67. $(25x^2y)^{\frac{1}{2}}$ 68. $(50ab^4)^{\frac{1}{2}}$ 69. $(16a^5b^6)^{\frac{1}{4}}$
 70. $\frac{(x^5y^6)^{\frac{1}{2}}}{z^{-\frac{1}{2}}}$ 71. $\frac{5^{\frac{1}{2}}a^{\frac{3}{2}}}{4^{\frac{1}{2}}}$ 72. $(\frac{-32x^{10}}{y^4})^{\frac{1}{5}}$ 73. $\frac{8^{\frac{1}{2}}a^{\frac{5}{2}}b^{\frac{3}{2}}}{(27c^4)^{\frac{1}{6}}}$

In 74–82, write each expression as a power with positive exponents in simplest form.

74. $(\frac{2a^{\frac{1}{2}}}{3a^{\frac{1}{6}}})^6$ 75. $(\frac{x^2y}{3x^4b^2})^{\frac{2}{3}}$ 76. $(\frac{4a^4b^6}{25a^{-1}b})^{\frac{1}{2}}$
 77. $(\frac{8a^2z^6}{27x^9a^{-4}z^{-1}})^{\frac{1}{3}}$ 78. $\sqrt{x^2y} \cdot \sqrt{x^4y^3}$ 79. $\frac{\sqrt[6]{a^5}}{\sqrt[5]{a^5}}$
 80. $\frac{\sqrt[3]{11x^5y^4}}{\sqrt{2x^5y^2}}$ 81. $\frac{\sqrt[5]{48xy^2}}{\sqrt[3]{6x^2y^4}}$ 82. $(\sqrt{2xy^2})(\sqrt[4]{16x^2y})$

83. Verify that the laws for power of a product and power of a quotient are true for the following examples. In each example, evaluate the left side using the rules for radicals and the right side using the rules for fractional exponents:

a. $(\sqrt[3]{27} \cdot \sqrt[3]{3})^2 \stackrel{?}{=} (27^{\frac{1}{3}} \cdot 3^{\frac{1}{3}})^2$ b. $(\frac{\sqrt[3]{3}}{\sqrt[3]{9}})^3 \stackrel{?}{=} (\frac{3^{\frac{1}{3}}}{9^{\frac{1}{3}}})^3$